Predicting Pedestrian Injury Metrics Based on Vehicle Front-end Design

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Abstract. Simulations provide vehicle designers with the capability to evaluate the safety of their designs in a wide variety of scenarios. However, the high-fidelity simulations required for safety assessment carry significant computational costs. As such, the engineering team must carefully select automotive designs to simulate, and use the results obtained to accurately predict the performance of new designs over a wide range of metrics. This paper describes the modeling of automotive simulation outputs to accurately predict a large number of widely used pedestrian injury metrics given the vehicle front-end design. The models in this paper allow the vehicle designer to identify and focus on the variables that most affect the different injury metrics, and determine which variables are most important to the overall safety performance of the vehicle.

1 Introduction

Designing the front-end of a vehicle to minimize the risk of injury to a pedestrian in a collision is a complex task faced by vehicle designers. The large number of design variables that can be adjusted and the fact that each design variable typically has a continuous range of values that it can take on results in a combinatorial number of potential designs. Vehicle designers have turned to modeling and simulation as a way of effectively and efficiently testing and evaluating font-end designs. Even though modeling and simulation does not require physically building the proposed design before testing it, the models can be computationally costly, sometimes taking up to a week to run a single simulation. As such, it is preferable for the designer to fully understand the effect that different design variables have on a pedestrian when the vehicle is involved in a front-end collision with a pedestrian, so that only those designs that are potentially viable are investigated.

This work looks at the task of accurately predicting commonly used pedestrian injury metrics given a specific vehicle front-end design, and determining which front-end design variables play a significant role influencing the injury metrics. In particular, there are 24 different design variables that can be adjusted, and 44 different injury metrics that can be recorded for a given design.

Thus the problem is to accurately predict the 44 different injury metrics given a set of 24 specific design variable values. Building accurate predictive models for this purpose will allow the search space to be effectively and efficiently searched for a design that satisfies multiple competing criteria (in addition to considering the pedestrian impact, vehicle designers must also consider aerodynamics, aesthetics, etc.). This paper begins with a literature review, which is followed by a discussion of the data and the methods used to build the predictive models. Results are then presented which show the success of these methods on a specific data set.

2 Literature Review

Much research has been conducted to understand the relationship between vehicle front-end design and pedestrian injuries. Niederer and Schlumpf (1984) analyze how four hood models affect pedestrian kinematics at different impact speeds and claim that both vehicle front geometry and stiffness influence pedestrian head impacts. In particular, they find that front shape dominates the gross motion, while the deformability affects the acceleration level during the direct contact. Similarly, Han et al. (2012) study the effects of vehicle impact speed and front design on pedestrian injury risks. They notice that effects on pedestrian body regions are mostly influenced by variation in vehicle front designs. Moreover, Han et al. (2012) conclude that the minicar has the best front-end geometry in terms of lowering the overall injury risk. Besides the primary impact with the car, pedestrians may also suffer from the secondary impact with the ground. Crocetta et al. (2015) investigate the role of vehicle front-end design when considering pedestrian-ground contact. Their results show that once the vehicle impact speed exceeds 40 km/h, low front vehicles like sedans are no longer advantageous in reducing the severity of head-ground impact (Crocetta et al., 2015, p. 68). These studies all confirm that vehicle front geometry is a vital factor to consider in pedestrian safety.

The idea of optimization has been applied in studies that aim to mitigate the negative effects on pedestrians involved in a collision. Using the nonparametric Radial Basis Function (RBF) to implement response surfaces, Zhao et al. (2010) developed an optimal vehicle front-end geometry to protect pedestrians' heads. By validating values obtained from the RBF model through simulations, Zhao et al. (2010) conclude that the RBF-based response surface model handles the non-linearity in head injury criterion (HIC) scores well, with strong predictive capability (p. 149). Likewise, Kausalyah et al. (2014) looked at lowering the risk of head injuries to both child and adult pedestrians. The authors first propose separated front designs for adults and children, as children are more likely to be run over by cars. By using genetic algorithms (GAs) to find the optimal solution and multinomial logistic regression (MLR) to deal with run-overs, Kausalyah et al. (2014) produce an integrated front end design that effectively reduces the HIC values for both types of pedestrian. The two studies represent common research interests on reducing pedestrian head impacts, as head injuries are mostly fatal (Crandall et al., 2002). Lower extremities are also well studied, since they are also commonly injured regions (Crandall et al., 2002). Lv et al. (2015) examine the reliability design optimization for designing an optimal vehicle front-end structure so as to minimize pedestrian lower extremity injury risks. By using the multi-objective particle swarm optimization (MOPSO) algorithm to incorporate probabilistic bounds into the optimization problem, the authors are able to capture uncertainty in design variables.

3 Simulation Setup

A multi-body human model and a vehicle model were used to simulate the vehicle impacting the pedestrian. The pedestrian model was developed by combining the upper body parts from MADYMO's scalable 50th percentile human male occupant model (ver 4.10, TNO) and the lower extremity part from Kerrigan (2008) and Hall (1998). The bio-fidelity of the pedestrian model was improved using cadaveric blunt impact test data (Rawska et al, 2015). A parametric vehicle model was developed using variables to characterize its geometry and structural stiffness. Four vehicle regions (bumper, grill, hood, and windshield, see Fig. 1) were defined using five landmarks. The definition of the eight variables representing the geometry of the vehicle followed those of Mizuno et al. (2005), and the ranges of the variables were selected to represent sedans, hatchbacks, and sport utility vehicles. The contact stiffness of each of the four regions was characterized using four parameters, namely p1, p12, F1, and F12, for a total of 16 variables (see Fig. 1). The ranges for each of the 24 design variables can be found in Tab. 1.

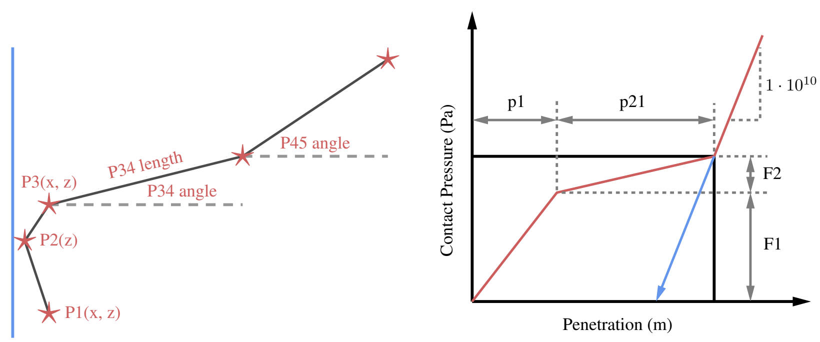

Fig. 1. Vehicle geometry and stiffness design variables

Each simulation required that each of the 24 different design variables (8 geometry related variables and 16 stiffness related variables) be assigned a specific value. Due to the size of the search space and the computational cost of each simulation, a Latin Hypercube sample of the design space was obtained using the maximinLHS() function in R. The resulting sample is a set of 1000 24-by-1 unique vectors, where each vector contains the specific value for each one of the

Table 1. Ranges for the 24 different design variables

Variable	Design Variable	Parameter	Units	Lower Bound	Upper Bound
v_1	P1(x)	Bottom depth	mm	0.0	95.3
v_2	P1(z)	Bottom height	mm	197.6	520.0
v_3	P2(z)	UBRL height	mm	521.2	667.6
v_4	P3(x)	Bonnet leading	mm	20.5	210.4
v_5	P3(z)	BLERL height	mm	703.9	1111.0
v_6	P34L	Bonnet length	mm	814.6	1474.1
v_7	P34 angle	Bonnet angle	$^\circ$	7.5	19.0
v_8	P45 angle	Windscreen angle	$^\circ$	21.0	42.1
v_9	F1	Bumper	kPa	0.5	6.0
v_{10}	p1	Bumper	mm	10.0	20.0
v_{11}	F21	Bumper	kPa	0.0	6.0
v_{12}	p21	Bumper	mm	20.0	100.0
v_{13}	F1	Grill	kPa	6.0	10.0
v_{14}	p1	Grill	mm	20.0	60.0
v_{15}	F21	Grill	kPa	0.0	2.0
v_{16}	p21	Grill	mm	0.0	40.0
v_{17}	F1	Hood	kPa	0.5	4.0
v_{18}	p1	Hood	mm	10.0	40.0
v_{19}	F21	Hood	kPa	0.0	5.0
v_{20}	p21	Hood	mm	20.0	100.0
v_{21}	F1	Windshield	kPa	0.05	2.0
v_{22}	p1	Windshield	mm	10.0	40.0
v_{23}	F21	Windshield	kPa	0.0	2.0
v_{24}	p21	Windshield	mm	20.0	200.0

24 design variables. The simulation described above was run 1000 times, where on each run a different one of the input vectors was used.

Given an input vector, the simulation produced peak values of 44 raw physical quantities which are widely used as injury metrics. These responses are categorized by the 10 body regions presented in Table 2. For each one of these 44 outputs, there are 1000 values corresponding to the 1000 unique input vectors. These 44 data sets containing 1000 data points each are what the analysis is performed on.

Table 2. Physical quantities used as injury metrics, grouped by body region

Location	Metrics
Head	HIC 15 (Eppinger et al., 1999) BrIC (Takhounts et al., 2013) Linear & Angular Accleration NIJ (Tension only) (Eppinger et al., 1999)
Face	Contact force (Cormier et al., 2011)
Neck	NIJ (Lund, 2003)
Thorax	Half Cmax at 4 levels (Viano et al., 1989) VCmax at 2 levels
Abdomen	VCmax VCmaxCmax
Pelvis	Lateral Force (Viano et al., 1989)
Thigh	Bending Moment at proximal, mid, and distal shafts (Kerrigan et al., 2004)
Knee	Angle
Leg	Bending Moment at proximal, mid, and distal shafts (Kerrigan et al., 2004)
Ankle	Angle Force(Funk et al., 2002)

4 Methodology

4.1 Data

There was minimal correlation between the 24 predictor (design) variables. The 44 response variable data sets could be characterized by 4 different types of distributions, namely skewed left, skewed right, normally distributed, and 'single' value. The 'single' value distribution is one in which the vast majority of data points are clustered around a single value, while the remaining minority are outliers. Examples of the four different types of response variable data set distributions can be seen in Figure 2 (skewed left: thorax_L_lowest_VCMax, skewed right: femur_R_dist, normally distributed: ankle_R_ang, 'single' value: abdomen_F_low).

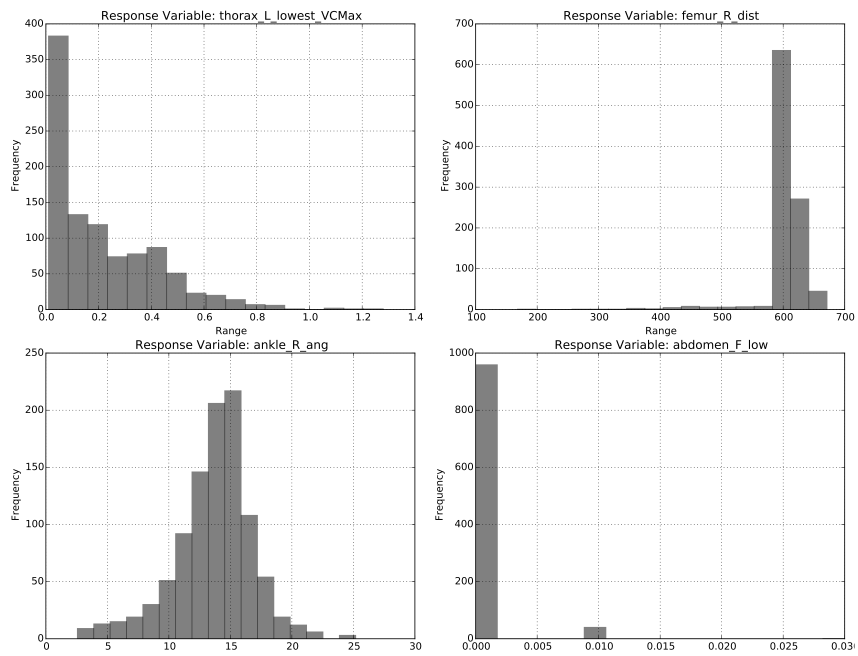

Fig. 2. Examples of the four different types of response variable distributions

4.2 Modeling Approach

The modeling approach randomly divides each response variable data set into a training set and a test set. This process is repeated multiple times, so that for a given response variable, the data set composed of 1000 values generates multiple (random) training and test sets. This approach gives the mean and standard deviations of both model goodness of fit metrics and model performance metrics.

Model Goodness of Fit Metric. The model goodness of fit metric is the traditional R^2 value in the case of the linear regression models, and the 'pseudo'-R^2 value in the case of random forest regression models. The 'pseudo'-R^2 value is calculated as

$$1 - \frac{\text{MSE}}{\text{Var}(y_{\text{train}})} \tag{1}$$

where MSE is the mean squared error and $\text{Var}(y_{\text{train}})$ is the variance of the response variable training set.

Model Performance Metric. The metric to judge the predictive performance of a model is the normalized root mean squared error (NRMSE). NRMSE is calculated as

$$\sqrt{\frac{\text{MSE(Model)}}{\text{MSE(Null Model)}}} \tag{2}$$

where the Null Model predicts the average response variable value of the training set ($\bar{y}_{\text{train}}$) regardless of the setting of the predictor variables. The Null Model is a minimum information model and forms a benchmark against which the performance of other (hopefully better) models can be evaluated. Given the definition of NRMSE, the closer the value of NRMSE is to zero for a given model, the better the model being evaluated is. For example, a Model with a NRMSE value of 0.89 corresponds to a MSE ratio value of 0.80 and thus has a MSE that is 20% less than the Null Model MSE. For the results presented later, a model was considered to be "good" if it had a NRMSE value of 0.90 or less.

Models Used. Due to the fact that the 44 different response variable data sets have starkly different distributions, the approach taken here is to model each response variable using both linear regression models and random forest regression models, and pick the best.

For the linear regression models four different groups of predictors were tried. The first group of predictor tried, group_1 predictors, consisted of 324 different predictors: the 24 original predictors, 24 squared predictors (obtained by squaring the 24 original predictors), and 276 predictors of the 2-way interactions between each of the 24 original predictors (obtained by simple multiplication of the 24 original predictors). The group_3 predictors were obtained by taking the natural log of the group_1 predictors, the group_4 predictors were obtained by taking the square root of the group_1 predictors, and the group_5 predictors were obtained by squaring the group_1 predictors. For each response variable a linear regression model was built using the lm() function in R. From that model, only predictor variables that had a p-value less than 0.1 were kept and were used to build a stepped linear regression model using the step() function in R.

For the random forest regression models a single group of predictors was tried, group_2 predictors. The group_2 predictors consist of 120 different predictors: the

24 original predictors, 24 squared predictors (squaring the 24 original predictors), 24 square root predictors (taking the square root of the 24 original predictors), 24 log predictors (taking the natural log of the 24 original predictors), and 24 reciprocal predictors (taking the reciprocal of the 24 original predictors). For each response variable a random forest regression model was built using the randomForest() function in R.

For the response variable data, four different transformations were considered: none (the data was left alone), the square root transformation, the natural log transformation, and the squaring transformation. Thus for each response variable there were 16 different linear regression models built (4 groups of predictors and 4 types of response variable transformations), and 4 different random forest models built (1 group of predictors, and 4 types of response variable transformations).

4.3 Generating the Training and Testing sets

In general a single training and testing set pair for a particular response variable was generated as follows. A random sample of size 700 data points, along with their associated predictor variable vectors, was taken from the data set of 1000. The testing set was then composed of the remaining 300 data points and their associated predictor variable vectors. However, for those response variables whose data sets fell under the 'single' value distribution description, the training and testing set generation process was slightly different.

Training and Testing Sets for Highly Skewed Data. Taking the response variable abdomen_F_low as an example, the histogram in the lower right hand corner of Figure 2 illustrates that the vast majority (959 data points) are zero and there are a few (41) outlier/extreme points that are non-zero.

For response variables like this, the training set was constructed by taking the n extreme data points and their associated predictor variable vectors. A set of 3n extreme jittered data points and jittered predictor variable vectors were created using the jitter() function in R. A random sample of 2*3n non-extreme data points and their accompanying predictor variable vectors together with the 3n jittered data points formed the training set so that the training set composition was one-third of jittered extreme data points and two-thirds non-extreme data points. The testing set was constructed by taking the n extreme data points and their associated predictor variable vectors together with the 1000-n-2*3n non-extreme data points and associated predictor variable vectors that were NOT used in the training set. The testing set has 1000-2*3n data points. For the response variable abdomen_F_low which has 41 extreme data points, the training set had 369 data points (123 extreme jittered and 246 non-extreme), while the testing set had 754 data points (41 extreme and 713 non-extreme).

5 Results

5.1 Predictive Modeling

Table 3 contains the overall results of the experimentation.

Table 3. Best model overall for each response variable

Response Variable	Model Type	Predictor Group	Resp. Trans.	R^2	NRMSE
thorax_L_low	original_rf	group_2	none	0.9661 (0.0017)	0.1915 (0.0134)
thorax_L_lowest	stepped	group_4	none	0.9729 (0.0020)	0.2014 (0.0116)
abdomen_R_highest	stepped	group_4	sqrt	0.9685 (0.0020)	0.2222 (0.0111)
thorax_L_lowest_VCMax	original_rf	group_2	sqrt	0.9496 (0.0024)	0.2343 (0.0152)
thorax_L_high	original_rf	group_2	none	0.9270 (0.0038)	0.2874 (0.0202)
neck_L	skewed_rf	group_2	sqrd	0.8944 (0.0133)	0.3369 (0.0682)
thorax_R_lowest_VCMax	skewed_rf	group_2	sqrt	0.9457 (0.0058)	0.3369 (0.0220)
head_5	skewed_rf	group_2	sqrd	0.8951 (0.0263)	0.3390 (0.1205)
pelvis_L	skewed_rf	group_2	sqrt	0.8459 (0.0255)	0.3911 (0.1220)
thorax_L_highest	original_rf	group_2	none	0.8672 (0.0101)	0.3987 (0.0311)
thorax_R_low_VCMax	skewed_rf	group_2	sqrt	0.9208 (0.0058)	0.3999 (0.0247)
thorax_R_low	stepped	group_1	none	0.9023 (0.0080)	0.4036 (0.0343)
thorax_R_lowest	original_rf	group_2	none	0.8705 (0.0085)	0.4044 (0.0360)
abdomen_L_highest	original_rf	group_2	sqrt	0.8503 (0.0066)	0.4346 (0.0245)
pelvis_R	original_rf	group_2	sqrt	0.8493 (0.0072)	0.4369 (0.0247)
head_2	skewed_rf	group_2	sqrd	0.8723 (0.0353)	0.4551 (0.1455)
thorax_L_low_VCMax	original_rf	group_2	sqrt	0.8002 (0.0142)	0.4966 (0.0590)
thorax_R_high	stepped	group_4	none	0.8195 (0.0102)	0.5761 (0.0404)
face_R	original_rf	group_2	sqrt	0.7744 (0.0096)	0.5788 (0.0326)
face_L	original_rf	group_2	sqrt	0.7743 (0.0099)	0.5791 (0.0319)
tibia_L_prox	stepped	group_1	log	0.7642 (0.0177)	0.6605 (0.0426)
knee_R	skewed_rf	group_2	sqrt	0.6335 (0.0828)	0.6632 (0.1760)
thorax_R_highest	original_rf	group_2	sqrt	0.6915 (0.0137)	0.7005 (0.0329)
abdomen_F_low	skewed_rf	group_2	log	0.8554 (0.0095)	0.7020 (0.0400)
femur_L_dist	stepped	group_1	sqrd	0.7223 (0.0221)	0.7073 (0.0622)
abdomen_F_high	original_rf	group_2	sqrt	0.6960 (0.0129)	0.7310 (0.0358)
ankle_R_force	skewed_rf	group_2	sqrd	0.7966 (0.0320)	0.7396 (0.0794)
tibia_L_mid	stepped	group_1	log	0.6930 (0.0281)	0.8052 (0.0618)
ankle_L_force	skewed_rf	group_2	sqrd	0.7043 (0.0884)	0.8740 (0.0503)
ankle_L_ang	original_rf	group_2	none	0.5625 (0.0241)	0.9489 (0.0600)

Each row contains:

1. The name of the response variable,
2. The type of model: (a) 'initial' for an initial linear regression model, (b) 'stepped' for a stepped linear regression model, (c) 'original_rf' for a random forest regression model that used the regular training and testing sets, or (d) 'skewed_rf' for a random forest regression model that used the training and testing sets for highly skewed data
3. The type of predictor used (one of "group_1", "group_2", "group_3", "group_4", or "group_5"),
4. The type of transformation applied to the response variable data: (a) "none", (b) "sqrt" for the square root transformation, (c) "sqrd" for the squared transformation, or (d) "log" for the natural log transformation,
5. The mean and standard deviation (in parenthesis) of the R^2 (for linear regression models) or pseudo-R^2 (for random forest regression models) goodness of fit metric, and

6. The mean and standard deviation (in parenthesis) of the NRMSE model predictive performance metric.

The table shows the best model for each response variable if the average NRMSE value is less than 1, where the "best" model is defined as the one that has the smallest average NRMSE value. The table is sorted in ascending order based on the mean NRMSE column.

Of the 44 different response variables, the table shows that there are 29 response variables that have "good" models, i.e., models where the average NRMSE is less than 0.9. What the table does not illustrate is that for many of the response variables, those that had a "good" linear regression model tended to also have a good random forest model. Thus if insights for a particular response variable need to be gained from a linear regression model, but the "best" model is deemed to be a random forest model, a linear regression model can be constructed that will also likely be "good".

5.2 Variable Importance

In addition to predicting new response values, the "best" random forest regression models can also be used to estimate the importance of the 24 original predictor (design) variables. This is important to designing the front end of the car as it allows the car designers to focus on specific design variables if they are interested in minimizing the effect of specific types of injuries.

The randomForest() function in R provides a measure of variable importance, %IncMSE. For a given predictor variable the metric indicates how much MSE increases when that variable is randomly permuted. The more important a variable is to the model, the more the model will degrade (i.e., the more MSE will increase) when that variable is randomly permuted. Thus the larger the value of %IncMSE for a variable is in a model, the more important that variable is to the model.

Because group_2 predictor variables are used in the random forest regression models, there are five different predictor variables to each one of the 24 design v_i, namely a) original variable, $v_{i,\mathrm{orig}}$, b) the squared version of the variable, $v_{i,\mathrm{sqrd}}$, c) the square root version of the variable, $v_{i,\mathrm{sqrt}}$, d) the natural log version of the variable, $v_{i,\mathrm{log}}$, and e) the reciprocal version of the variable, $v_{i,\mathrm{recip}}$. For a particular training and testing set t, the %IncMSE for design variable v_i is

$$mse\_inc(v_{i,t}) = \sum_{j \in \{\text{orig, sqrd, sqrt, log, recip}\}} \frac{mse\_inc(v_{i,t,j})}{5} \tag{3}$$

The average %IncMSE for design variable v_i over all training and testing sets t $\in$ T is

$$mse\_inc(v_{i,t}) = \sum_{t \in T} \frac{mse\_inc(v_{i,t})}{|T|} \tag{4}$$

Figure 3 is a bar plot that shows the average %IncMSE calculated for each design variable v_i in the random forest models for response variable thorax_L_low.

The average %IncMSE is calculated as outlined above, with $|T|$=100. The 8 geometry related design variables are colored yellow, while the 16 stiffness related design variables as colored gray. This plot allows the reader to quickly identify a) which type of design variables (geometry or stiffness) are most important to predicting the response variable, and b) the relative importance of each design variable. For instance Figure 3 shows that while geometry design variables are more important than stiffness variables, it also illustrates that by far design variables v4 and v5 are the most important variables to the model.

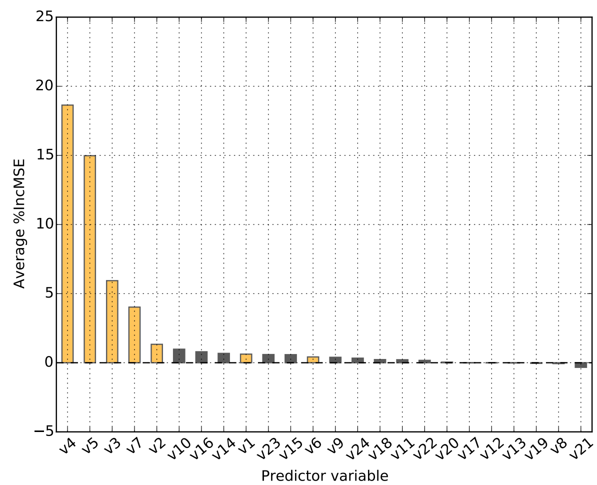

Fig. 3. Variable importance for response variable thorax_L_low

Although individual variable importance plots provide a sense of which design variables are important to predicting specific response variable values, they do not show which design variables are important overall (i.e., over all response variables under consideration). This can be achieved by averaging the average %IncMSE value for each design variable v_i over all response variables. Doing so yields Figure 4, which shows the overall design variable importance plot. This figure illustrates that in general the designers should focus primarily on the geometry related variables (specifically v4 and v5 and to a lesser extent v7 and v3), and also some specific stiffness related variables (perhaps v12, and v16).

6 Conclusion

The task of designing the front-end of a vehicle is a complex and time-consuming process. Modeling and simulation have been adopted in order to reduce the time and effort required to test and evaluate a single proposed design. This research used modeling and simulation to build a data set which was used to build predictive models. These models predict various pedestrian injury metrics given a specific car design without the need for physically building and testing the design, or even simulating the design (which itself can be computationally intensive).

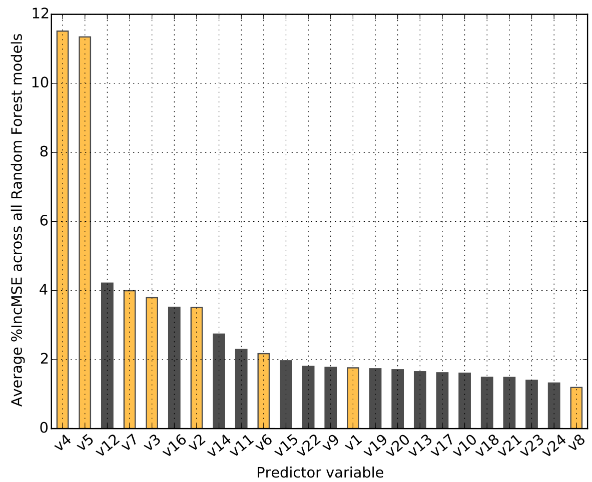

Fig. 4. Overall variable importance

Our work does have some limitations. There are still response variables for which the current methods do not yield "good" models (models where NRMSE < 1). Further research is required to determine how best to address these response variables. Given the good predicitve models, the next step is optimization – using the predictive models to search the design space for promising front end designs.

This work is the first step in a process where the output from an initial run of simulations is used to build good predictive models. In turn, these predictive models will guide the next run of simulations, and help the vehicle designer focus only on those designs that have potential to be the final design. Overall the results from this research will help reduce the ever increasing cost of designing a car to meet, among other criteria, the required safety standards.

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